AN APPROACH TO DIRECTIONAL WAVELET CONSTRUCTION AND THEIR USE FOR IMAGE COMPRESSION

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Abstract: *In this paper we construct a new type of directional wavelet transform based on lifting scheme and Quincunx sampling. Based on known separable wavelet predictors we build nonseparable 2D wavelet predictors with directional properties. As example we use resulting wavelet transform for image compression for motion blurred images.*

Key words: *directional wavelets, lifting scheme, quincunx sampling, image compression*

1. INTRODUCTION

Wavelets [1] became popular in few past years in mathematics and digital signal processing area because of their ability to effectively represent and analyze data. Typical application of wavelets in digital signal processing is image compression. (Mostly lossy case) [1]. Because of their multiresolution signal representation they are the best candidates for progressive transmission coding (e.g. SPIHT [3]). There are many generalizations of original orthogonal wavelet systems [1, 4]. New means to perform Wavelet transform (WT) offers Lifting scheme [2]. We create novel type of 2D WT with strong directional properties from 1D prototype using Lifting and recursive quincunx sub sampling. We use it for image compression using SPIHT algorithm and discuss the results.

1.1 LIFTING SCHEME

Originally was Lifting scheme developed to improve properties of biorthogonal wavelets. Later was proved that every wavelet can be factorized into lifting steps [2]. This realization has many advantages such: speed up, in place calculation, simple treatment of boundaries, possibility of nonlinear operations and irregular lattices, invertibility. Classical implementation of WT uses two-band filter bank (FB) with recursion on its low pass (LP) [1]. In lifting implementation [2] is signal partitioned into even and odd components that are then mutually predicted by t_i (to zero signal in HP part) and updated by s_i (see Fig. 1).

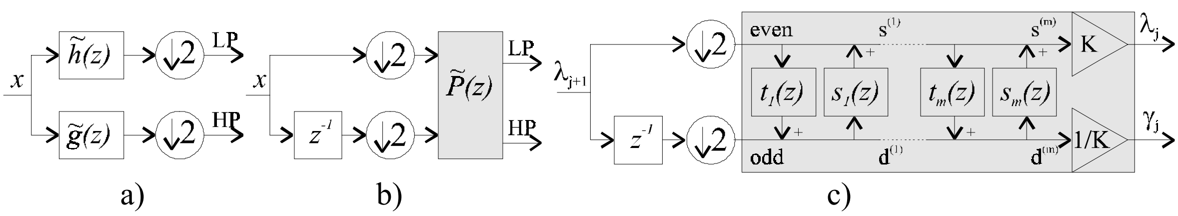


Fig. 1. Analysis part of the 2-band filter bank: a) classical case b) polyphase representation with delay c) factorised polyphase representation to lifting steps

After normalization is lifting algorithm recursively applied to LP part. Backward transform simply undoes all ladder steps from right to left using reversed operators.

1.2 2D WAVELET TRANSFORM AND QUINCUNX

Effective and natural way to realize WT in two dimensions is to apply 1D transform to rows and columns of signal.

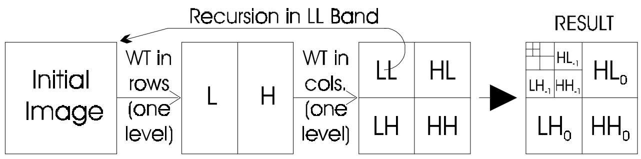


Fig. 2. Nonstandard decomposition in 2D wavelet transform

Most used is non-standard decomposition (Fig. 2), where basis functions are then tensor products of 1D bases [1]. Realizing 2D WT via lifting, coefficients can be reordered analogically.

Another approach is to direct design two-dimensional wavelet filters as in [5]. On rectangular lattice this correspond to 4-band filter bank. Our approach uses Quincunx sampling lattice [1] and to obtain LL band as in Fig. 2 completes decomposition of one level in two phases as shown in Fig. 3. Similar construction was presented also in [4, 5].

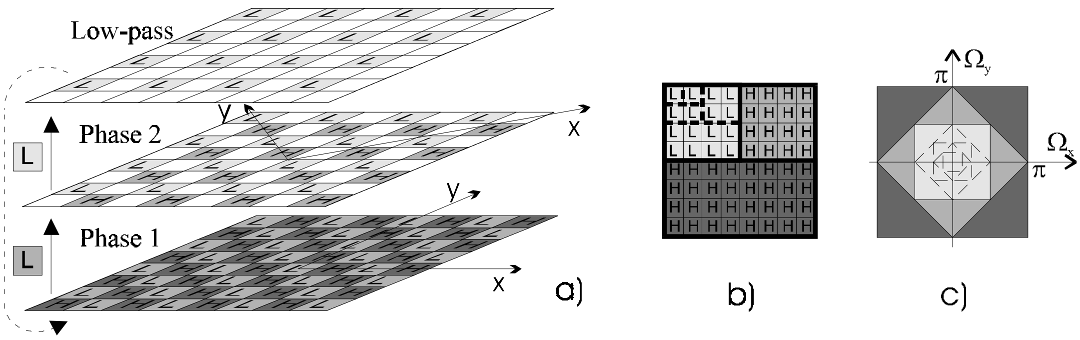


Fig. 3. 2D WT using Quincunx: a) lattices and their recursion b) Coefficient reordering c) Distribution of frequency bands

In Phase1 we predict/update coefficients on Quincunx lattice with the rest of them. In Phase 2 we use Low pass output L of Phase 1 and similarly predict/update but with lattice rotated by 45°. LL band consisting low pass coefficients L is further recursively decomposed. Corresponding signal partitioning to frequency bands when used ideal half band filters is in Fig. 3c. After transform we can reorder coefficients according to Fig. 3b, which is representation more suitable for compression methods.

In predict/update phases can be used 2D neighborhood instead of 1D ones. In [4] authors computed 2D Neville filters (filters that implement polynomial interpolation) of various order on Quincunx lattice.

2. DIRECTIONAL WAVELETS

When using separable wavelets for coding, the coding error is spread vertically and horizontally. Transform decorrelation ability is based only on this two directions too. Non-separable wavelets allow more précised design of directional properties of transform process. So we can design e.g. isotropic wavelets or directional wavelets with best decorrelation properties in given direction. These wavelets are more suitable for such images when we expect stronger correlation in some direction (for example in motion blurred images), there are directional wavelets more suitable from 2 points of view:

a) Computational effectiveness - most time is spend to decorrelation in given direction

b) Visual properties - coding error is spread in desired direction

Using concept of Lifted wavelet transform on Quincunx lattices introduced in section 1.2 we can effectively implement non-separable directional wavelets.

To test this approach we constructed directional predictors in 8 basic directions for use in lifting steps on Quincunx lattices (see Fig. 4). Example basis functions are shown in Fig. 5. Predictors are constructed from interpolating filter with order 4 that is Deslauries-Dubuc wavelet with 4 vanishing moments DD (4,4). In construction is important to:

1) Preserve the predictor energy, i.e. sum of all coefficient should be always 1
2) Maintain a minimal decorrelation property in non interesting directions, i.e. in at least one Lifting phase should be used 2D predictor (not rotated 1D predictor)
3) Maintain directional properties on Image boundaries, i.e. there should be implemented special mirroring that preserves the predictor direction.

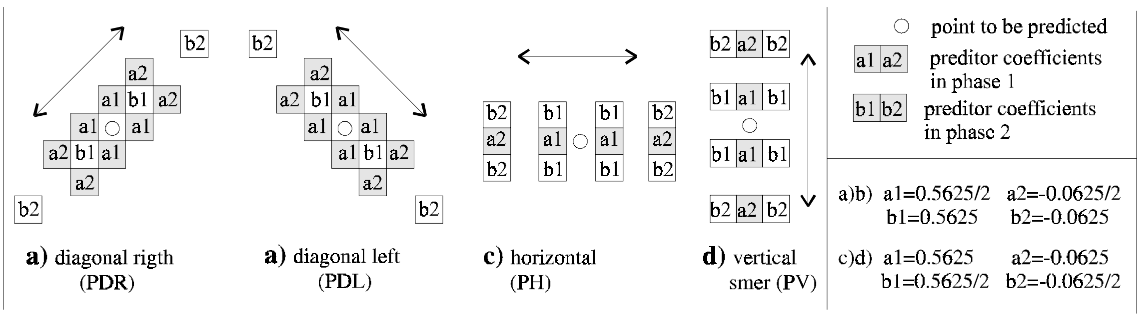
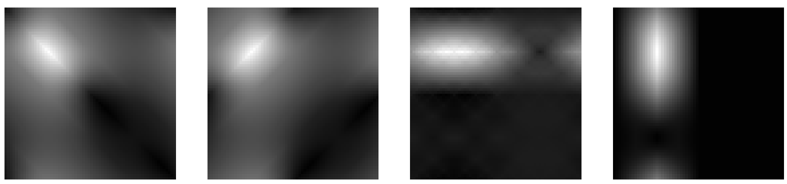


Fig. 4. Suite of 2D Predictors for 8 basis directions, based on DD (4,4) wavelet



a) PDL b) PDR c) PH d) PV

Fig. 5. Examples of basis functions (2^{nd} in x and y direction) of directional wavelets

2.1 COMPRESSION RESULTS

To test the developed directional wavelets effectiveness we implemented them into SPIHT image compression algorithm. As a test image we take directionally blurred Lena gray scale image 256x256, 8bpp. The results depend on correlation properties of an unblurred image version (for example Lena has biggest correlation in vertical, then in diagonal right direction).

Examples of achieved results are shown in Table1 and Fig. 6. As we see the directional wavelet can outperform the originating wavelet in the sense of PSNR, while preserving better the visual characteristics of an original image.

Bit rate [bpp]		2	0.5	0.1	0.01	Image blur direction
PSNR[dB]	PDR	53.29	45.41	35.76	23.3	Diagonal
	DD(4,4)	52.92	44.64	34.51	22.45	
PSNR[dB]	DD(4,4)	53.72	46.3	38.01	24.58	Diagonal
	LDR	52.5	46.11	37.94	24.34	

Table 1. Example compression results for different bit rates using blurred Lena image

a) Original (blur diagonal right) b) DD(4,4) c) PDR

Fig. 6. Reconstructed image (original (blurred diagonal right), compression ratio 1:400 using SPIHT algorithm and b) separable wavelet transform c) directional non-separable transform

3. CONCLUSION

In this paper we showed an approach to non-separable directional wavelet transform construction, which can provide an effective way to decorrelate or compress certain type of images, for example motion blurred images. We demonstrated the effects when used simple directional wavelet set in image compression using SPIHT algorithm. Building bigger sets of directional wavelets and introducing mechanism to adaptively choose the best one for given image (or image part) can further extend the effectiveness of presented method.

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