Compression of bitonal images using nonlinear wavelet transform

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Abstract

The capability, to effectively store and transmit bitonal images is still a challenge in the data compression area. We propose a method for coding of bitonal images based on nonlinear wavelet transform approach. The transform is a "N to N" reversible mapping based on Haar transform and offers wavelet-like data decorrelation, but without expanding the dynamic range of the input signal in the transformed domain. We show the effectivity of this representation for static image compression using SPIHT algorithm compared to classical discrete wavelet transform.

1. Introduction

The transformation step in image compression using transform coding, leads generally to dynamic range expansion. That means the number of possible output coefficients is greater than the number of possible input values. This effect can be very easily observed when using the discrete wavelet transform. Dynamic range expansion presents some problems. If the transform is being performed when a channel width is limited to *n* bits, some data loss will result due to forced truncation of the coefficients. If custom hardware implementing the transform is to be designed and built, circuitry for handling the extra bit(s) adds complexity and cost to the design. Because modern computer architectures allocate storage in 8–bit increments, an implementation of the transform on a computer will require 16 bits to store each 9–bit coefficient.

Many currently available non-expansive methods experiment with Haar transform as the basis. Some of them [1],[2] are discontinuous and therefore can't be used for lossy compression. We present an own non-expansive Haar-like transform called NEHaar that preserves some of the Haar transform properties. We compare our method against the classical Haar transform.

2. Haar transform

The Haar transform can bee seen as a function of a pair of input values A,B – point coordinates, that performs a rotation of that point about the origin in Euclidean (or $L2$) space. In this space, points in the domain that are equidistant from the origin lie on a circle. Given two input values A and B the corresponding output values, high–pass value H and low–pass value L *(called also spectral coefficients),* can be computed by equations (1) and (2).

$$L = \frac{A+B}{\sqrt{2}} \qquad H = \frac{A-B}{\sqrt{2}} \tag{1}$$

Or in a matrix form:

$$\begin{bmatrix} L \\ H \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} A \\ B \end{bmatrix} \tag{2}$$

We can see that if the $A, B \in \{-N...N\}$, then $L, H \in \{-\frac{2N}{\sqrt{2}} ... \frac{2N}{\sqrt{2}}\}$, that means the transform domain values are expanded in the range.

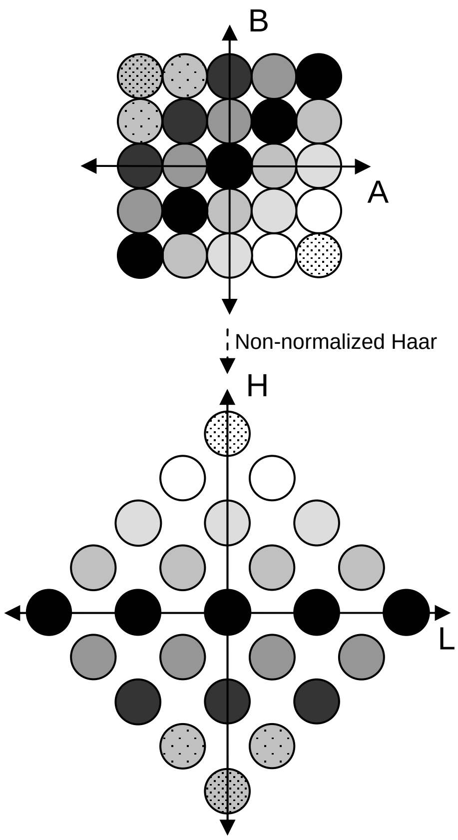


Figure 1. Haar transform (non-normalized)

The situation can be seen on figure 1.

Currently the progressive compression algorithms, such as SPIHT [4] offer one of the best compression ratios when using lossy compression. The reconstructed coefficients obtained using such a schema are *L',H'* whereby:

$$0 \le |L'| \le |L| \text{ and } 0 \le |H'| \le |H| \tag{3}$$

Assuming that the sign of the coefficient is always transmitted as first by the progressive encoding schema, then using obtained approximations *L',H'* of *L,H* we can obtain approximations *A',B'* of the original input values *A,B*. Using Haar transform, we always satisfy following condition:

$$\begin{aligned} A' < B' &\text{ when } A < B \\ A' > B' &\text{ when } A > B \end{aligned} \tag{4}$$

Condition (4) ensures that the two image pixels will not be inverted in magnitude after reconstruction and the magnitude order of the two neighboring image pixels is always either preserved or at least not compromised (the pixels can become equal). Magnitude inversion causes disturbing artifacts in the reconstructed image. This effect is more obvious when having more levels in the DWT spectrum. The condition assumes that spectral coefficients' sign is transmitted, which is often case for most of the spectral coefficients in the spectrum, when using progressive encoding schemas. The condition validity can be simply followed by choosing a part of the space and inspecting the transformed values. Figure 2. is such a case:

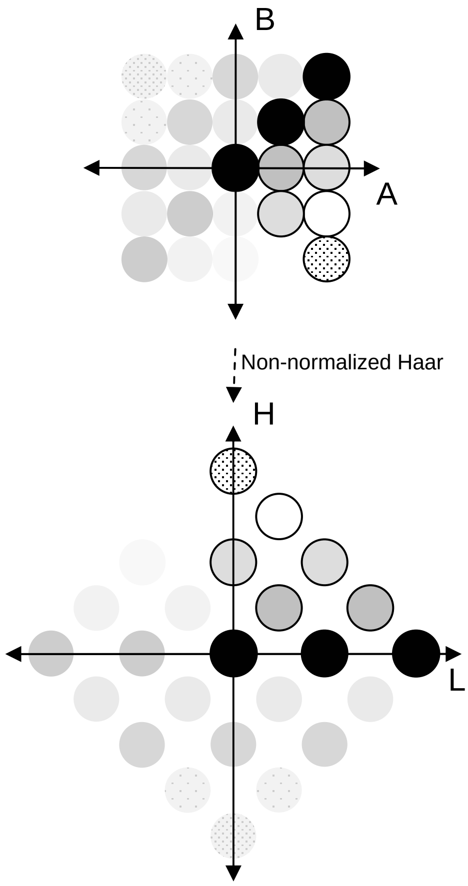


Figure 2. Magnitude order preservation in Haar transform

One can see, that the highlighted values occupy a part of the original domain where:

$$A \geq B \text{ and } A \geq 0 \qquad (5)$$

They are transformed into the first quadrant of the transformed domain. When satisfying (3) (and sign transmission), the transformed values will not leave the first quadrant of the transformed domain and therefore the (4) will be satisfied in the original domain.

3. NEHaar transform

The NEHaar transform is a rotation in L2 space in as it shown on the figure 3.:

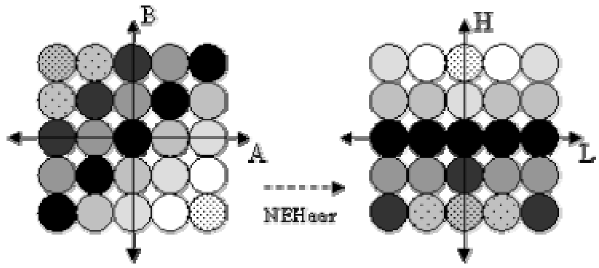


Figure 3. NEHaar transform

It is a non-expansive "N to N" solution. The transform satisfies the condition (4) and therefore it can be used together with progressive encoders, since it preserves the magnitude order of the input values. The transform is a "more dense" version of Haar transform with data decorrelation capability:

$$|A| \to |B| \;\; \Rightarrow \;\; |H| \to 0 \qquad (6)$$

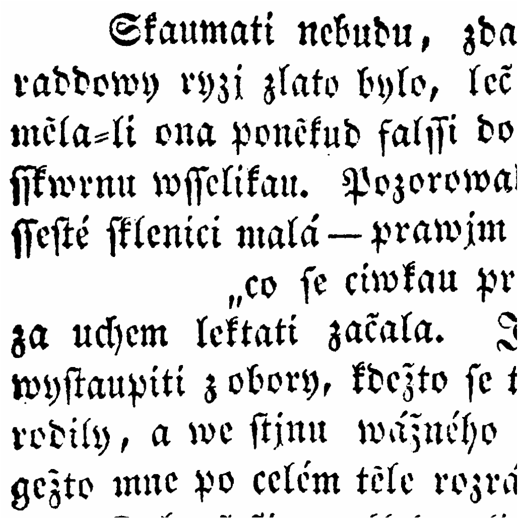
It works similar to Piecewise–Linear Haar (PLHaar) presented in [3], but it is derivated direct from the Haar transform itself and we were able to couple it with SPIHT encoder.

4. Compression results

We compared the NEHaar with the Haar transform. NEHaar was realized as a LookUpTable (LUT). Transformed images were compressed using SPIHT compression algorithm. At present state, we observed that the transform is not suitable for coding all group of images, since, as it can be seen on the Figure 3., the DP parts are favorized in the mapping process and have a higher gain as it is with Haar transform. This leads to brightness changes in the image. The Haar transform is resistant against these effects, but at the cost of redundancy in the transformed domain. We observed that if having lower level of color planes, this is not a problem anymore and the transform behaves well when using bitonal images.

Table I. Compression results [PSNR]

Schema / Bitrate	Haar	NEHaar
1	35.91	31.99
0.5	25.27	18.95
0.25	18.85	12.10
0.1	15.91	10.91
0.05	14.01	9.97


Skaumati nebudu, zda
raddowy ryzi zlato bylo, leč
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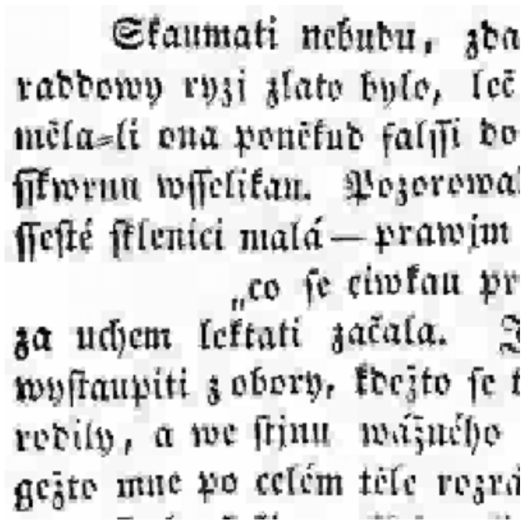
za uchem lektati začala. Z
wyſtaupiti z obory, kdežto ſe t
rodily, a we ſtjnu wážného
gežto mne po celém těle rozrá

Figure 4. Original image

As you can see in Table I., the new method does not outperforms the Haar transform in sense of PSNR, but the overall visual quality is better, since the edges are not distorted. As the original image we used a black/white text (see Figure 4.). The reconstructed text can be seen on Figures 5a,5b.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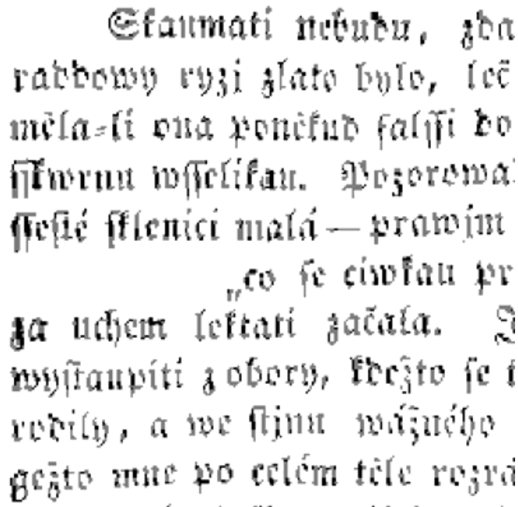
a)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b)

Figure 5. Reconstructed text at 0.05 bpp. a) Haar transform, b) NEHaar transform

5. Conclusion

We presented nonlinear wavelet transform modification. In the current state it is suitable for bitonal images compression. The transform itself is realized using LUT, but in fact it is a rotation in the space like the Haar transform. Moreover, since we have a mapping within finite number of symbols, new postprocessing possibilities arise, since we can expect lower number of output symbols.

Using analytical formula, this process could lead to "*N to N*" transform for *M* input values. We expect then the reconstruction error, to redistribute into the spectral coefficients more effectively as it is with *M=2*, so the reconstructed image will not show artifacts when using a higher lever of color planes.

References:

[1] H. Chao, P. Fisher, and Z. Hua. An approach to integer wavelet transformations for lossless image compression. In Chen, Li, Micchelli, and Xu, editors, Advances in Computational Mathematics, volume 202 of Lecture Notes In Pure and Applied Mathematics, chapter 2, pages 13–38. Marcel Dekker, Inc., 1999.

[2] J. G. Senecal, M. A. Duchaineau, and K. I. Joy. Reversible n–bit to n–bit integer Haar–like transforms. In Proceedings of the 7th IASTED Conference on Computer Graphics and Imaging, 2004.

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[4] A. Said, Pearlman, W., A. A New Fast and Efficient Image Codec Based on Set Partitioning in Hierarchical Trees, IEEE Transactions on Circuits and Systems for Video Technology, vol. 6, pp. 243-250, June 1996