The First Erlang Formula and Traffic Description in Asynchronous Networks

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Abstract - The paper deals with the possibility of the 1st Erlang formula utilization in Asynchronous Transfer Mode (ATM) networks. This statement is possible on the basis of research of statistical features of traffic source input flow. We consider ATM network type where in the case of Virtual Path important parameters are loss, link utilization and bandwidth. The evidence of the 1st Erlang formula utilization in asynchronous networks is achieved through the model of video source traffic. We have used the discrete Markov chain to create the video source traffic.

Keywords - 1st Erlang formula, ATM network, Virtual Path, Traffic Model

I. INTRODUCTION

It is obvious that with the help of the 1st Erlang formula, where such parameters as loss \( B \), link utilization \( \rho \) and bandwidth \( c \) rise, we are able to describe the traffic in synchronous networks. On the other hand, to describe the traffic in asynchronous networks of Asynchronous Transfer Mode (ATM) [14] network type Markov chains [1], [2], [9], [11] are often used. But their disadvantage is that they are very complicated and at the same time they demand complicated estimations. Papers [3], [4] also deal with the idea of the utilization of 1st Erlang formula. As in the case of ATM networks the optical fiber is used as transmission medium, it is possible to consider ATM networks to be networks with low losses. ATM is technology that provides communication environment for voice, data and video services. Networks with low losses are characterized by the 1st Erlang formula which expresses the connection loss probability in virtual path (VP) [5], [6]. The question if the utilization of the 1st Erlang formula is possible in asynchronous networks rose from this statement. In this paper we are going to deal with the research of the utilization of the 1st Erlang formula in asynchronous networks.

The 1st Erlang formula, which is suitable for the research of its utilization in asynchronous networks, is known in the following form:

\[
\frac{\rho^c}{\sum_{k=0}^{c} \frac{\rho^k}{k!}} = B
\]  

(1)

where \( \rho \) represents percentage utilization of link, \( c \) is link bandwidth and \( B \) is link loss.

In next chapter we are going to deal with the research of parameter dependencies \( B, \rho \) and \( c \).

II. DEPENDENCY RESEARCH AMONG PARAMETERS

The aim is to estimate dependencies among \( B, \rho, c \) parameters on the basis of the 1st Erlang formula in such a manner that always one parameter will keep constant size and the dependency among other two parameters will be observed. Estimations are done on the basis of (1).

A. Dependency Among the Link Utilization and Bandwidth at constant Loss

On the basis of 1st Erlang formula it is possible to get dependency of link utilization \( \rho \) from bandwidth independent at constant loss \( B \). Results are shown in fig. 1. For the purpose of comparison the dependency of link utilization from bandwidth at loss 10% and 5% has been estimated. From the fig. 1 it is visible that with the increasing bandwidth it is possible to increase the link utilization the way that loss is preserved. The narrower the loss requirement (i.e. parameter \( B \) gets smaller value), the less link utilization is possible.

B. Dependency Among the Link Utilization and Bandwidth at constant Loss

On the basis of the 1st Erlang formula we get the loss dependency \( B \) from the link utilization \( \rho \) at constant bandwidth \( c \). Results are shown in fig. 2.
Estimations were done for the constant bandwidth $c = 4$ Mbit/s, 10 Mbit/s and 50 Mbit/s. From fig. 2 it is visible that with the increasing link utilization for given bandwidth the loss increases as well. Further it is visible that at the same link utilization $\rho$ and at different values $c$ the loss is different. With the increasing bandwidth at the same link, utilization loss decreases.

Fig. 1. Dependency of link utilization from bandwidth of given loss

Fig. 2. Loss dependency from the link utilization at constant bandwidth

C. Dependency Among the Loss and Bandwidth at constant Link Utilization

On the basis of the 1st Erlang formula we get the loss dependency $B$ from the bandwidth $c$ at constant link utilization $\rho$. Results are shown in fig. 3.

Fig. 3. Loss dependency from the bandwidth at constant link utilization

- Transition matrix $P = [P_{ij}]$.
- Sizes’ vector of video source model state $s = [s_1, \ldots, s_M]$.

Where $M$ expresses number of states in Markov chain and $s_1, \ldots, s_M$ video source model states are representing rate of cell generation.

As the result we have video source model for $M=7$. 

\[
\begin{bmatrix}
0.7888 & 0.1472 & 0.0439 & 0.0171 & 0.0028 & 0.0003 & 0 \\
0.4178 & 0.4595 & 0.0976 & 0.0214 & 0.0031 & 0.0005 & 0 \\
0.7256 & 0.2451 & 0.0185 & 0.0091 & 0.0017 & 0 & 0 \\
0.7877 & 0.1872 & 0.0219 & 0.0011 & 0.0021 & 0 & 0 \\
0.7933 & 0.1733 & 0.0233 & 0.0100 & 0 & 0 & 0 \\
0.7500 & 0.2500 & 0 & 0 & 0 & 0 & 0 \\
0 & 1.0000 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

\[\begin{align*}
s_1 &= 26.873 \text{ kbit/s} \quad s_4 = 189.924 \text{ kbit/s} \\
s_2 &= 75.711 \text{ kbit/s} \quad s_5 = 241.409 \text{ kbit/s} \\
s_3 &= 135.687 \text{ kbit/s} \quad s_6 = 302.333 \text{ kbit/s} \\
s_7 &= 389.520 \text{ kbit/s}.
\]

For the purposes of simulation necessary simulation model consisting of ATM node without buffer store and with video traffic source [10], [12] on input of ATM node has been created. Input video traffic sources have different requirements from the point of view of bandwidth, loss and link utilization. There is an output link on the other end of ATM node, which has parameters defined as follows: bandwidth, loss requirement and link utilization. All these parameters were adjusted according to requirements stemming from requirements of particular situations.

III. UTILIZATION OF THE FIRST ERLANG FORMULA IN ASYNOCHRONOUS NETWORK

The aim is to prove the possibility of the 1st Erlang formula’s utilization in asynchronous networks on the basis of simulations with the help of designed mathematical model. There were parameters from real video source used during the creation of model for video traffic source, which is shown in [7] for video Ice Age DVD.

On the basis of the method shown in [8] parameters necessary for the video source model creation are estimated as follows:
A. Dependency Among the Link Utilization and Bandwidth at Constant Loss

Dependency of link utilization $\rho$ from bandwidth $c$ (512 kbit/s, 1024 kbit/s, 1544 kbit/s, 2048 kbit/s and 4096 kbit/s) at constant loss $B$ (1% to 5%) has been simulated. Results are recorded in table I and fig. 4.

Table I and fig. 4 verify that with the increasing bandwidth it is possible to increase link utilization the way that loss is preserved. The narrower the loss requirement, the less link utilization is possible.

<table>
<thead>
<tr>
<th>$B$ [%]</th>
<th>$c$ [kbit/s]</th>
<th>$\rho$ [%]</th>
<th>$B$ [%]</th>
<th>$c$ [kbit/s]</th>
<th>$\rho$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>512</td>
<td>60.53</td>
<td>5</td>
<td>512</td>
<td>77.49</td>
</tr>
<tr>
<td></td>
<td>1024</td>
<td>75.4</td>
<td></td>
<td>1024</td>
<td>87.21</td>
</tr>
<tr>
<td></td>
<td>1544</td>
<td>80.2</td>
<td></td>
<td>1544</td>
<td>91.4</td>
</tr>
<tr>
<td></td>
<td>2048</td>
<td>83.19</td>
<td></td>
<td>2048</td>
<td>93.94</td>
</tr>
<tr>
<td></td>
<td>4096</td>
<td>89.53</td>
<td></td>
<td>4096</td>
<td>96.78</td>
</tr>
</tbody>
</table>

B. Dependency Among the Loss and Link Utilization at Constant Bandwidth

Dependency of loss $B$ from link utilization $\rho$ at constant bandwidth $c$ = 512 kbit/s, 1024 kbit/s and 2048 kbit/s has been simulated and results are recorded in table II and fig. 5.

It is obvious that with the increasing link utilization for given bandwidth the loss increases as well. Further it is visible that with the same link utilization $\rho$ and at different values $c$ the loss is different as well. With the increasing bandwidth the loss decreases.

<table>
<thead>
<tr>
<th>$c$ [kbit/s]</th>
<th>$\rho$ [%]</th>
<th>$B$ [%]</th>
<th>$c$ [kbit/s]</th>
<th>$\rho$ [%]</th>
<th>$B$ [%]</th>
<th>$c$ [kbit/s]</th>
<th>$\rho$ [%]</th>
<th>$B$ [%]</th>
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<tr>
<td>512</td>
<td>50.8</td>
<td>0.29</td>
<td>1024</td>
<td>60.5</td>
<td>1.03</td>
<td>2048</td>
<td>69.5</td>
<td>2.57</td>
</tr>
<tr>
<td></td>
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<td>70.8</td>
<td>0.69</td>
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<td>77.5</td>
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<td></td>
<td>89.5</td>
<td>12.24</td>
<td></td>
<td>90.2</td>
<td>6.74</td>
<td></td>
<td>90.9</td>
<td>3.43</td>
</tr>
</tbody>
</table>

C. Dependency Among the Loss and Link Utilization Constant Bandwidth

Dependency of loss $B$ from bandwidth $c$ (512 kbit/s, 1024 kbit/s, 1544 kbit/s, 2048 kbit/s and 4096 kbit/s) at constant link utilization $\rho$ has been simulated. Results are shown in fig. 6. Fig. 6 verifies that at constant link utilization $\rho$ and increasing bandwidth the loss decreases.

Fig. 5. Loss Dependency from the Bandwidth at Constant Link Utilization

Further it is visible that with the same bandwidth and different link utilizations the loss is different. With the increasing link utilization the loss increases as well.

IV. RESULTS

Following tendencies were shown with the help of the 1st Erlang formula:
- with the increasing bandwidth it is possible to increase link utilization the way that the given transfer path loss is preserved,
- at constant bandwidth and increasing link utilization the transfer path loss increases as well,
- at constant link utilization and increasing bandwidth the transfer path loss decreases.

On the basis of simulations with the help of designed mathematical model of video source model (built-up with
In the case of dependencies among transfer path loss and link utilization at constant bandwidth just a small difference occurred. It is obvious from the comparison of fig. 2 and fig. 5. In the case of estimations with the help of the 1st Erlang formula sudden loss growth at smaller bandwidths occurred, whereas in the case of mathematical video source model just progressive growth of transfer path loss with progressive growth of link utilization occurred.

Consequently, simulation results verified possibility of the 1st Erlang formula utilization also in asynchronous networks, where such parameters as loss, bandwidth and link utilization are taken into consideration.

V. CONCLUSIONS

Today we are witnesses of rapid development in telecommunication field. Converged technologies and convergence processes are extremely attractive topic. Convergence represents evolution path of switch from actual telecommunications to future modern multimedia telecommunication infrastructure for landlines as well as for mobile networks. Next generation networks (NGN) are results of convergence.

There exist several important fields within the frame of NGN matter, which draw a lot of attention of professionals from the telecommunication field. We speak about improvement of QoS parameters [13], [15], [16], management systems, multimedia, routing matter in NGN networks, dimensioning of networks, optimization of system components of NGN platform and many other.

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