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An iterative algorithm for minimum cross entropy thresholding

C.H. Li *, P.K.S. Tam [1]

Department of Electronic Engineering, Hong Kong Polytechnic University, Hung Hom, Hong Kong

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Abstract

A fast iterative method is derived for minimum cross entropy thresholding using a one-point iteration scheme. Simulations performed using synthetic generated histograms and a real image show the speed advantage and the accuracy of the iterated version. © 1998 Elsevier Science B.V. All rights reserved.

1. Introduction

Image thresholding based on the gray level histogram is an efficient and important tool for image segmentation (Haralick and Shapiro, 1992). Various algorithms have been proposed to solve this problem. Among them, Li and Lee (1993) introduced the minimum cross entropy thresholding algorithm for thresholding by selecting the threshold which minimizes the cross entropy between the segmented image and the original image. In this paper, a fast iterative method is derived for the minimum cross entropy method.

2. Iterative method for minimum cross entropy thresholding

In this section, an iterative method to obtain the threshold that minimizes the minimum cross entropy will be derived. For a histogram h defined on the gray level range $[1,L]$, the zeroth and the first moments of the foreground and background portions of the thresholded histogram are respectively,

$$m_{0a}(t) = \sum_{i=1}^{t-1} h(i), \quad m_{0b}(t) = \sum_{i=t}^{L} h(i),$$
$$m_{1a}(t) = \sum_{i=1}^{t-1} ih(i), \quad m_{1b}(t) = \sum_{i=t}^{L} ih(i). \tag{1}$$

The portions' means are defined as

$$\mu_a(t) = \frac{m_{1a}(t)}{m_{0a}(t)}, \quad \mu_b(t) = \frac{m_{1b}(t)}{m_{0b}(t)}. \tag{2}$$

The minimum cross entropy method (Li and Lee, 1993) selects the threshold which minimizes the cross entropy of the image and its segmented version. The criterion function is found to be

$$\eta(t) = -m_{1a}(t)\log(\mu_a(t)) - m_{1b}(t)\log(\mu_b(t)). \tag{3}$$

* Corresponding author. E-mail: chli@en.polyu.edu.hk.

[1] E-mail: enptam@hkpucc.polyu.edu.hk.

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The optimal threshold t_{op} is given by the minimizer of Eq. (3),

$$t_{op} = \arg\min_t \eta(t). \tag{4}$$

The calculation of the optimal threshold involves the evaluation of $\eta(t)$ for all possible threshold values. The computation can be significantly reduced by developing a numerical method for the minimization. First, note that a necessary condition for the minimum of $\eta(t)$ is given by setting the derivative of $\eta(t)$ to zero. The derivative of $\eta(t)$ is

$$\eta'(t) = h(t)\left(t \log \frac{\mu_a(t)}{\mu_b(t)} - (\mu_a(t) - \mu_b(t))\right). \tag{5}$$

For $\eta'(t)$ to be zero, either $h(t) = 0$ or the second term in Eq. (5) is zero. As $h(t) = 0$ is satisfied only by those threshold values where the image does not contain such gray values, these solutions can be considered as trivial solutions to the thresholding problem. Thus the solution is sought at the zeros of the second term in Eq. (5). Setting the second term to zero and simplifying, we get

$$t = \frac{\mu_b(t) - \mu_a(t)}{\log(\mu_b(t)) - \log(\mu_a(t))}. \tag{6}$$

Applying the one-point iteration method to Eq. (6), we get the following iterative procedure for calculating the optimal threshold,

$$t_{n+1} = \mathrm{round}\left\{\frac{\mu_b(t_n) - \mu_a(t_n)}{\log(\mu_b(t_n)) - \log(\mu_a(t_n))}\right\}, \quad n \geqslant 0, \tag{7}$$

with an initial guess t_0 until the iteration converges. The convergence is the condition where $t_{n+1} = t_n$. The round(x) function rounds x to its nearest integer.

The existence of a solution to Eq. (6) can also be proved by noting the general theorem on solutions of the one-point iteration method (Atkinson, 1988). A solution exists within the interval $[1, L]$ where the histogram is defined if the following is satisfied

$$1 \leqslant \frac{\mu_b(t) - \mu_a(t)}{\log(\mu_b(t)) - \log(\mu_a(t))} \leqslant L. \tag{8}$$

Proof. Since the histogram is defined in the range $[1, L]$, $1 \leqslant \mu_a \leqslant L$ and $1 \leqslant \mu_b \leqslant L$. Without loss of generality, we can assume $\mu_b > \mu_a$. Thus $\mu_b = r\mu_a$ where $r > 1$. The second inequality in Eq. (8) can be written as

$$\mu_b\left(1 - \frac{1}{r}\right) \leqslant L\log(r),$$

or

$$\frac{\mu_b}{L} \leqslant \frac{\log(r)}{1 - \frac{1}{r}}.$$

As the right-hand side is always larger than one and the left-hand side is less than one, the above inequality is always satisfied.

The first inequality in Eq. (8) can be written as

$$\log(r) \leqslant \mu_b\left(1 - \frac{1}{r}\right),$$

or

$$\frac{1 - \frac{1}{r}}{\log(r)} \leqslant \mu_a.$$

As the right-hand side is always less than one and μ_a is always greater than or equal to one, the above inequality is always satisfied.

Therefore, a solution to Eq. (6) always exists. The computational savings and the accuracy of the iterative procedure will be investigated in the next section.

3. Results and discussions

The iterative minimum cross entropy thresholding procedure is tested on a number of synthetic histograms to see if the procedure gives solutions consistent with those using the exhaustive search. Each histogram is generated as a mixture of two Gaussian distributions with parameters $(\rho_1, \mu_1, \sigma_1)$ and $(\rho_2, \mu_2, \sigma_2)$ representing the proportions, means and standard deviations of the two Gaussians, respectively. Additive noise is also added to the Gaussians to generate more realistic histograms. The parameters of the Gaussians are obtained from the following condition:

- ρ_1 is uniformly sampled from the interval (0.01,0.99), $\rho_2 = 1 - \rho_1$,
- μ_1 is uniformly sampled from the interval (71.5,121.5),
- μ_2 is uniformly sampled from the interval (135.5,185.5),
- σ_1 are uniformly sampled from the interval (5,30), $\sigma_2 = \sigma_1$.

The intervals for μ_1 and μ_2 are chosen such that μ_1 and μ_2 are separated from each other and away from the maximum and minimum gray values of 255 and 0. The intervals for the standard deviations σ_1 and σ_2 are chosen to cover situations of minimal overlapping to high overlapping of gray levels between the foreground and the background. The proportions of the background against the foreground are in ratios ranging from 1:99 to 99:1, which should cover commonly occurring situations. A total of 1000 histograms are generated. Fig. 1 shows some of the generated histograms.

For each of the 1000 histograms, the threshold is obtained from both the iterative procedure and an exhaustive search. The difference in the threshold values between the iterative version and the exhaustive version is recorded for the 1000 histograms and the mean absolute difference and the standard deviation of the difference are selected as the average performance criteria for the iterative procedure. Other criteria of the iterative procedure are the average number of iterations and the standard deviation of the number of iterations. The results are shown in Table 1.

Table 1
Summary of performances of iterative procedure: (a) average absolute difference, (b) standard deviation of difference, (c) average number of iterations, (d) standard deviation of iterations.

	(a)	(b)	(c)	(d)
$t_0 = 128$	0.39	1.11	5.08	2.43
$t_0 = 64$	0.67	1.77	8.57	2.73

In the first set of results in Table 1, the iterative method is started using an initial threshold value of 128 for all of the 1000 histograms. Since 128 is the middle of the gray level range of the histogram, this value is a natural one for initializing the iteration. The average error for the iterative version is only 0.39, implying that the iterative method correctly locates thresholds in more than half of the testing histogram. The standard deviation is very close to

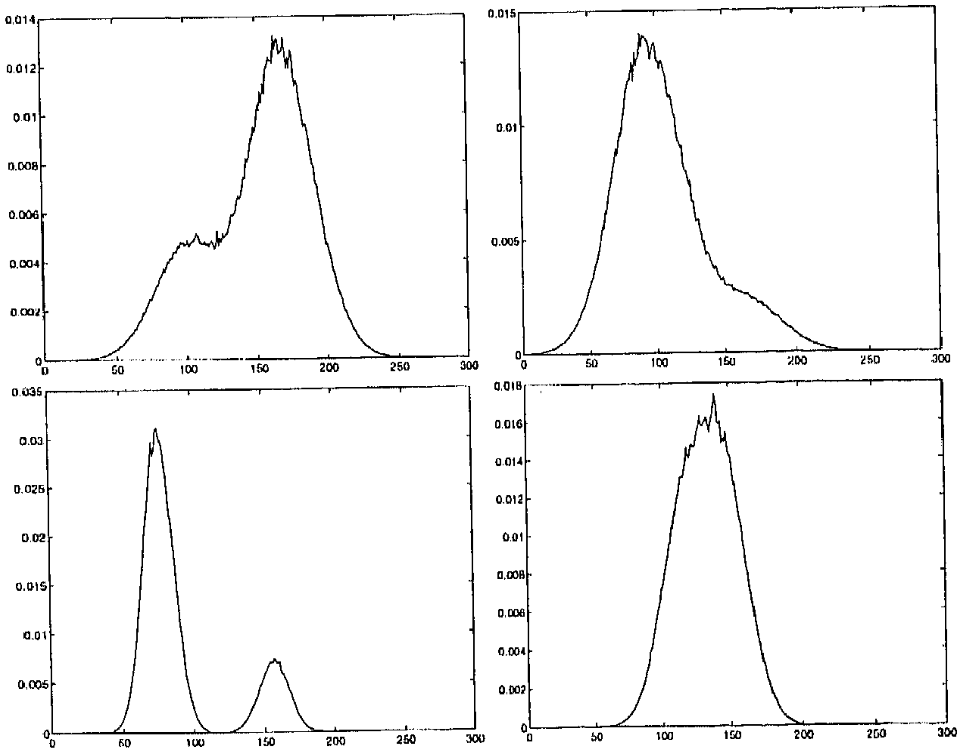

Fig. 1. Samples of synthetic histograms.

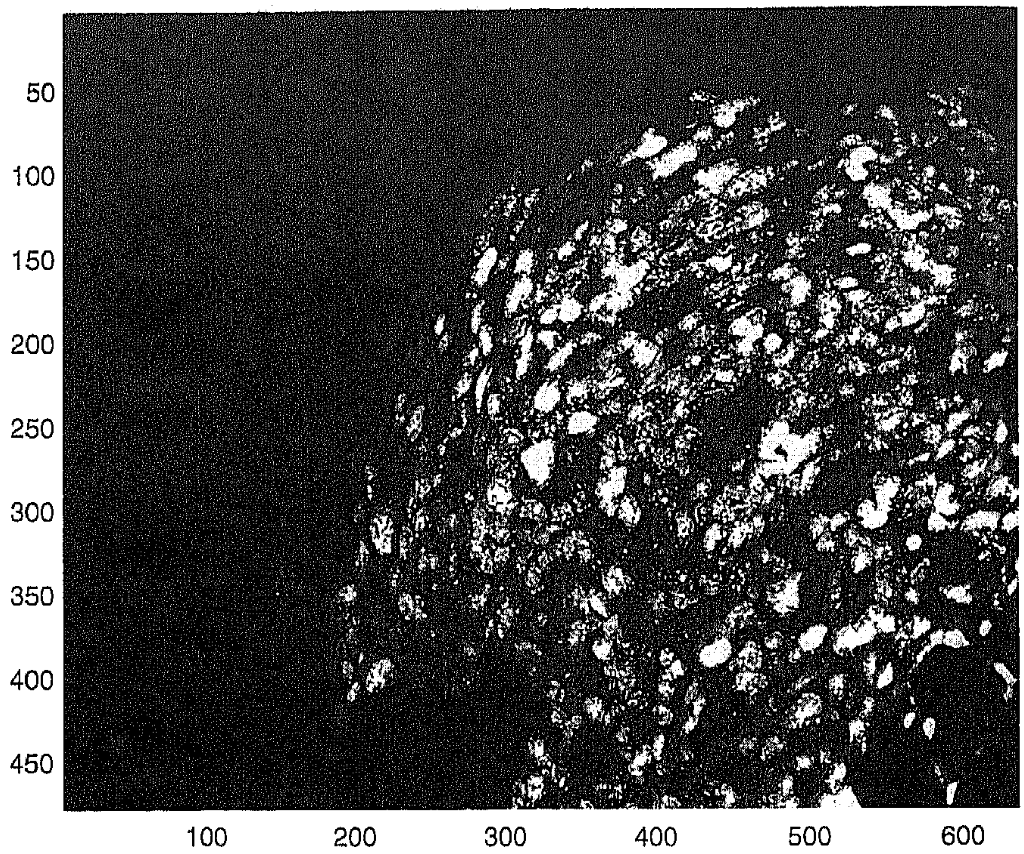

Fig. 2. Cell image.

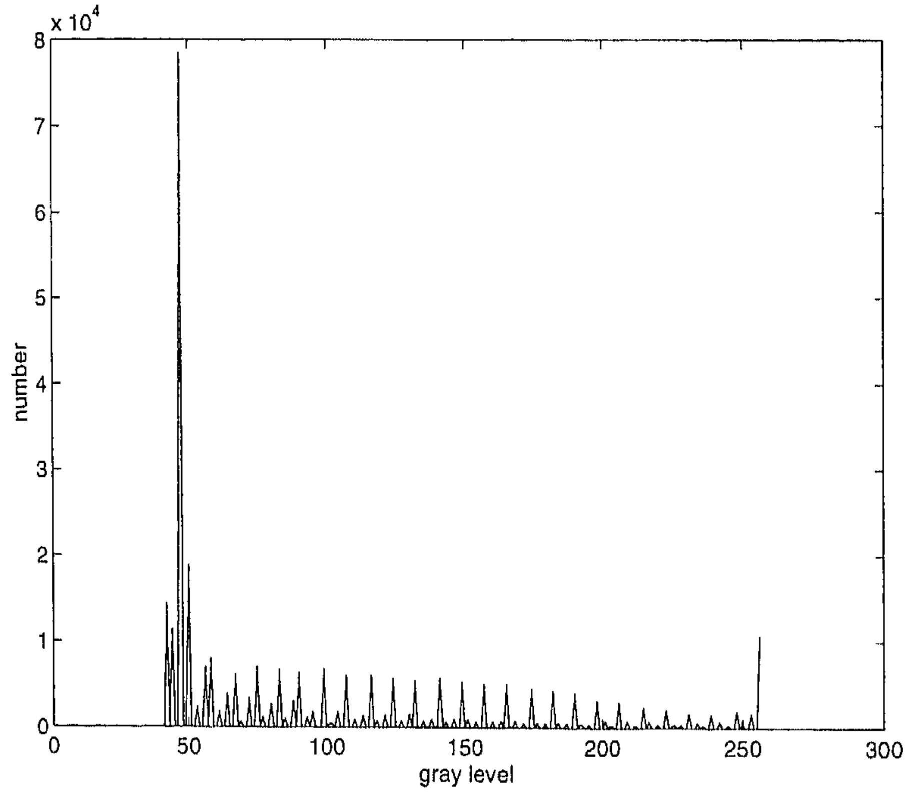

Fig. 3. Histogram of cell image.

one, implying that the errors of the iterative version do not fluctuate much. The average number of iterations is only five which means that the threshold can be obtained by calculating Eq. (7) five times on average. On the other hand, the exhaustive search method requires calculating Eq. (3) L times for a histogram with L levels where L is typically 256. Since the computational requirement for each calculation of Eq. (3) and Eq. (7) are similar, the computational saving of the iterative version is significant. The standard deviation of the number of iterations is less than 3; thus the variation in the number of iterations is not too large. Moreover, if an exact solution is needed, the iterative solution can serve as a initial search for the exact solution.

The iterative method is also repeated for the 1000 histograms using a fixed initial threshold value of 64. This experiment tests the performance of the algorithm by deliberately setting an initial threshold which is far from the locations of the correct thresholds. Referring to the results in the second column in Table 1, the average error in threshold increases to 0.67 which is still less than one. There is also an increase in the number of iterations needed to achieve convergence. Thus if the initial estimate is far from the correct one, more iterations are needed to attain convergence. However, the saving in computations is still significant compared with the exhaustive search method.

The iterative method for minimum cross entropy is also applied to a real image of stained cell nuclei; Fig. 2 shows the cell image and Fig. 3 shows the histogram of the cell image. As can be seen from the profile of the histogram, there is a large amount of noise in the histogram and no distinct valley can be found. By using the minimum cross entropy algorithm, the threshold is found to be 100. By using the proposed iterative minimum cross entropy algorithm, the threshold is found to be 104 using 5 iterations. The error associated with using the iterative version can be found by the finding the percentage area of the histogram portion between the two threshold values and is found to be 0.85%. Thus an error of less than one percent of the overall image is associated with using the iterated version in finding out optimal threshold, which is likely to be considered as

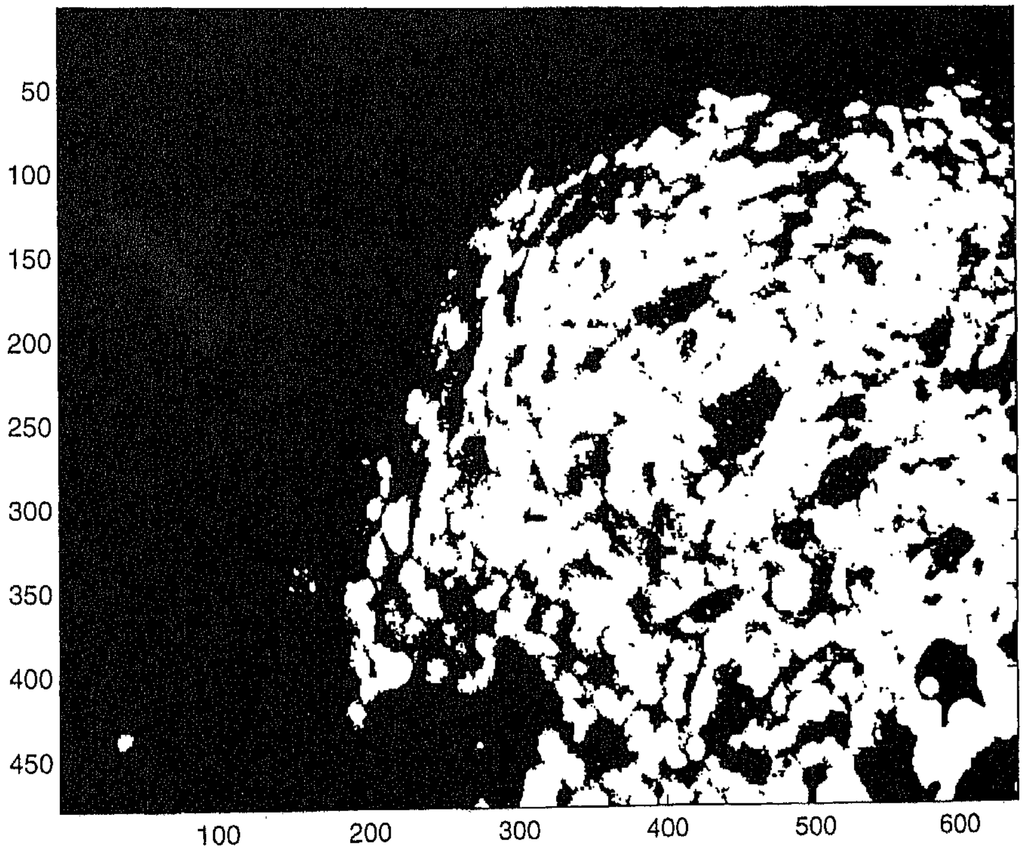

Fig. 4. Segmented cell image.

acceptable for most applications. The segmented result, using the threshold given by the iterated version is shown in Fig. 4. The result agrees with human expert evaluation.

Thus it is concluded that the iterative method for minimum cross entropy thresholding accurately locates the threshold while significantly reducing the computations required.

References

Haralick, R.M., Shapiro, L.G., 1992. Computer and Robot Vision. Addison-Wesley, Reading, MA.

Li, C.H., Lee, C.K., 1993. Minimum cross entropy thresholding. Pattern Recognition 26, 617–625.

Atkinson, K.E., 1988. An Introduction to Numerical Analysis. Wiley, New York.