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INVESTIGATIONS ON FUZZY THRESHOLDING BASED ON FUZZY CLUSTERING

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Abstract—Thresholding, the problem of pixel classification is attempted here using fuzzy clustering algorithms. The segmented regions are fuzzy subsets, with soft partitions characterizing the region boundaries. The validity of the assumptions and thresholding schemes are investigated in the presence of distinct region proportions. The hard k means and fuzzy c means algorithms have been found useful when object and background regions are well balanced. Fuzzy thresholding is also formulated as extraction of normal densities to provide optimal partitions. Regional imbalances in gray distributions are taken care of in region normalized histograms. © 1997 Pattern Recognition Society. Published by Elsevier Science Ltd.

Fuzzy clustering Thresholding Segmentation Fuzzy c means algorithm
Bayesian classifier

1. INTRODUCTION

Gray level thresholding is the process of partitioning pixels in a digital image into two mutually exclusive and exhaustive regions. Let $\mathscr{I} = [I_{mn}]_{M\times N}$ denote the image with pixel at (m,n) assuming a discrete gray value $I_{mn} \in \{0,1,\ldots,L-1\}$ defined over a universe $\mathbf{L} = [0, L-1]$. The problem of thresholding is that of identifying an optimal threshold T and segmenting the scene into two meaningful regions—object (O) and background (B), i.e.

$$O = \{I_{mn} \mid I_{mn} \geq T\}, B = \{I_{mn} \mid I_{mn} < T\}. \quad (1)$$

The geometrical and statistical characteristics of the histogram play an important role in threshold identification. Thresholding is a preferable method of segmentation when object and background are distinguishable using gray values alone. In such a scene, the perturbation of gray values around the mean values of object and background gray distributions provides a bimodal histogram and the threshold $T \in \mathbf{L}$ is considered as the valley point between the two modes so as to minimize the probability of misclassification. Due to the inherent uncertainties associated with the real life situations, identification of such a unique threshold becomes quite difficult and this led to the development of a number of algorithms based on objective functions whose maxima or minima correspond to the threshold T. Many of these criteria are based on region separability, entropy, moment, fuzziness, business, etc. Comprehensive surveys discussing various aspects of thresholding can be found in the literature.[1–3] Frequently, the threshold selection algorithms are compared with the help of experimental results on a set of real life images or on a set of histograms.[4,5] The major observations point to the importance of Otsu's[6] and minimum error thresholding[7] algorithms. Other popular methods employ the entropy[8] and moment[9] of the gray level histogram for global threshold selection.

Otsu[6] proposed a threshold selection scheme to maximize the class separability of object and background regions with the help of first- and second-order statistics. This scheme provides excellent results for a large class of real life images. Kapur's[8] method attempts to maximize the region normalized entropy of object and background regions so as to maximize the information in a segmentation process. This algorithm yields good results when object and background gray distributions are similar. The minimum error threshold selection scheme due to Kittler and Illingworth[7] is based on the assumption that the object and background gray values are normally distributed and is found to give excellent results even in the presence of a wide range of region scatters.

All these classical threshold selection algorithms classify the pixels deterministically into object and background classes and therefore fail to reflect the structural details embedded in the original gray distribution. It may also be seen that many of these schemes perform excellently well for a set of images depending on the underlying assumptions and yield poor results often in other situations. The defensive argument in favor of many of these *ad hoc* algorithms may be the psychovisual subjectivity of image segmentation. Even though the results provided by the experimental studies are valid, it is difficult to conclusively compare the merits and demerits of various algorithms without proper analytical explanations. When the regions are separable by gray values alone, thresholding can be defined as the process of identification of an optimal discriminant gray value and thereafter partitioning the scene to minimize the error in pixel classification. Therefore, the problem of thresholding should be attempted with the robust pattern classification tools.

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In view of the above discussions, we have investigated an appropriate threshold selection scheme which

(a) provides a soft thresholded description of the image based on fuzzy set theoretic concepts,

(b) can incorporate a wide range of object-background size and scatter imbalances, and

(c) does not depend excessively on assumptions of gray distributions in the image.

1.1. *Fuzzy sets and fuzzy thresholding*

Ever since its introduction to the literature,[10] fuzzy sets are finding extensive applications to describe the vague concepts in the modern mathematical framework. A modern and readable exposition to the theory of fuzzy sets may be found in reference (11). A fuzzy set $\tilde{A}$ is characterized by a membership function $\mu_{\tilde{A}}(x) \in [0, 1]$, which provides the compatibility or attachment of x to $\tilde{A}$. The classical set-theoretic concepts are found to be the special cases of fuzzy sets when the membership function reduces to a bivalent one, i.e. $\mu_{\tilde{A}}(x) \in \{0, 1\}$.

The classical pattern classification algorithms perform excellently when classes are clearly distinguishable. In a number of real life situations, however, such a hard demarcation is quite difficult. Instead of the conventional deterministic assignment of a sample to a class, fuzzy partitioning strategies provide soft description of the classes, where each of the sample points is assigned a membership in each of the classes. Such a partition reflects the identities of the samples and thereby preserves the structural details in the original data set. Application of fuzzy set-theoretic concepts to pattern recognition resulted in a number of elegant strategies to incorporate the ambiguity in measurement[12] as well as classification process.[13,14] Fuzzy clustering algorithms extracting complex geometrical structures in feature space are reported in literature.[13,14]

The concept of fuzzy partitioning can be extended for digital image thresholding by visualizing the object and background regions as fuzzy sets, $\tilde{O}$ and $\tilde{B}$, with each of the pixels showing a partial membership in each of the regions according to its gray value, i.e. $\mu_{\tilde{O}}(I_{mn}) \in [0, 1]$, $\mu_{\tilde{B}}(I_{mn}) \in [0, 1]$. With this sort of a partition, regions are no more guaranteed to be mutually exclusive; in other words, there may not exist a crisp boundary between regions.

A fuzzy thresholded description of an image can be characterized by two membership functions $\mu_{\tilde{O}}$ and $\mu_{\tilde{B}}$ in such a way that they reflect the nature of object and background gray distribution even after thresholding. The description reflects the compatibility measure of each of the pixel/gray value in object and background regions.

Pal[15,16] extended the definition of classical thresholding [equation (1)] to fuzzy setting by defining a membership function:

$$\begin{array}{ll} \mu_{\tilde{O}}(I_{mn}) < 0.5 & \text{if } I_{mn} < T, \\ \mu_{\tilde{O}}(I_{mn}) > 0.5 & \text{if } I_{mn} > T \end{array} \tag{2}$$

with the crossover of membership distribution coinciding with the hard threshold T. The membership distribution was identified with the help of an S function so as to minimize the index of fuzziness. Huang[17] assigned the memberships to the pixel with the help of the relationship between its gray value and mean gray value of the region to which it belongs. He assigned the membership with the help of the absolute difference between the gray value of the pixel and the mean gray value of the region to which it belongs. In this case, the image is viewed as a single fuzzy set where the membership distribution reflects the compatibility of the pixels to the region to which it belongs. Pal's method reflects the ambiguity in the transition region between object and background classes, while Huang's method depicts the geometrical structure of object and background gray densities.

2. THRESHOLDING: A CLUSTERING FORMULATION

2.1. *Assumptions*

Our first assumption is that the object and background regions are separable by gray value alone and the histogram is generated by the addition of gray density functions corresponding to object and background. We also assume that the "true" object and background gray values are perturbed by a physical process to form a continuous non-negative function $f(d(j, v_i), \sigma_i)$; $i = 1, 2$ of gray value $j \in \mathbf{L}$ with continuous derivative. This function monotonically decreases with d and characterizes the gray distribution of the region, where $\sigma_i \in \mathbf{R}$ controls the dispersion of the distribution. Unless otherwise stated, $d(j, v_i)$ is considered as the Euclidean distance in one dimension between v_i and j, i.e. $|v_i - j|$. From the symmetry of the distribution, it may be seen that v_i is the mean (or true) gray value of region i. Throughout the discussion, we interchangeably use the suffixes $i = 1$ and 2 in place of background (B) and object (O) regions.

The histogram representing the frequency of occurrence may be generated as

$$h_j = f(d(j, v_1), \sigma_1) + \rho f(d(j, v_2), \sigma_2), \quad j \in \mathbf{L}. \tag{3}$$

Here ρ corresponds to the ratio of sizes of object and background regions in the image, while another parameter $\gamma = (\sigma_2/\sigma_1)$ denotes the ratio of variances. When $\gamma = \rho = 1.0$, the object and background regions have equal size and equal dispersion. A typical example of f is a Gaussian function with a total number of N_i pixels, i.e.

$$f(d(j, v_i), \sigma_i) = \frac{N_i}{\sigma_i\sqrt{2\pi}} \exp\left[\frac{-d(j, v_i)^2}{2\sigma_i^2}\right]. \tag{4}$$

In Fig. 1(a), we show an example of object and background distribution and the histogram as a sum of these two in Fig. 1(b). Bayes decision theory provides us with a minimum error threshold at T_o where both object and background distributions provide equal gray density. The corresponding error in thresholding is shown as the shaded region in Fig. 1(a). It may be seen that the minimum error threshold corresponds to the valley of

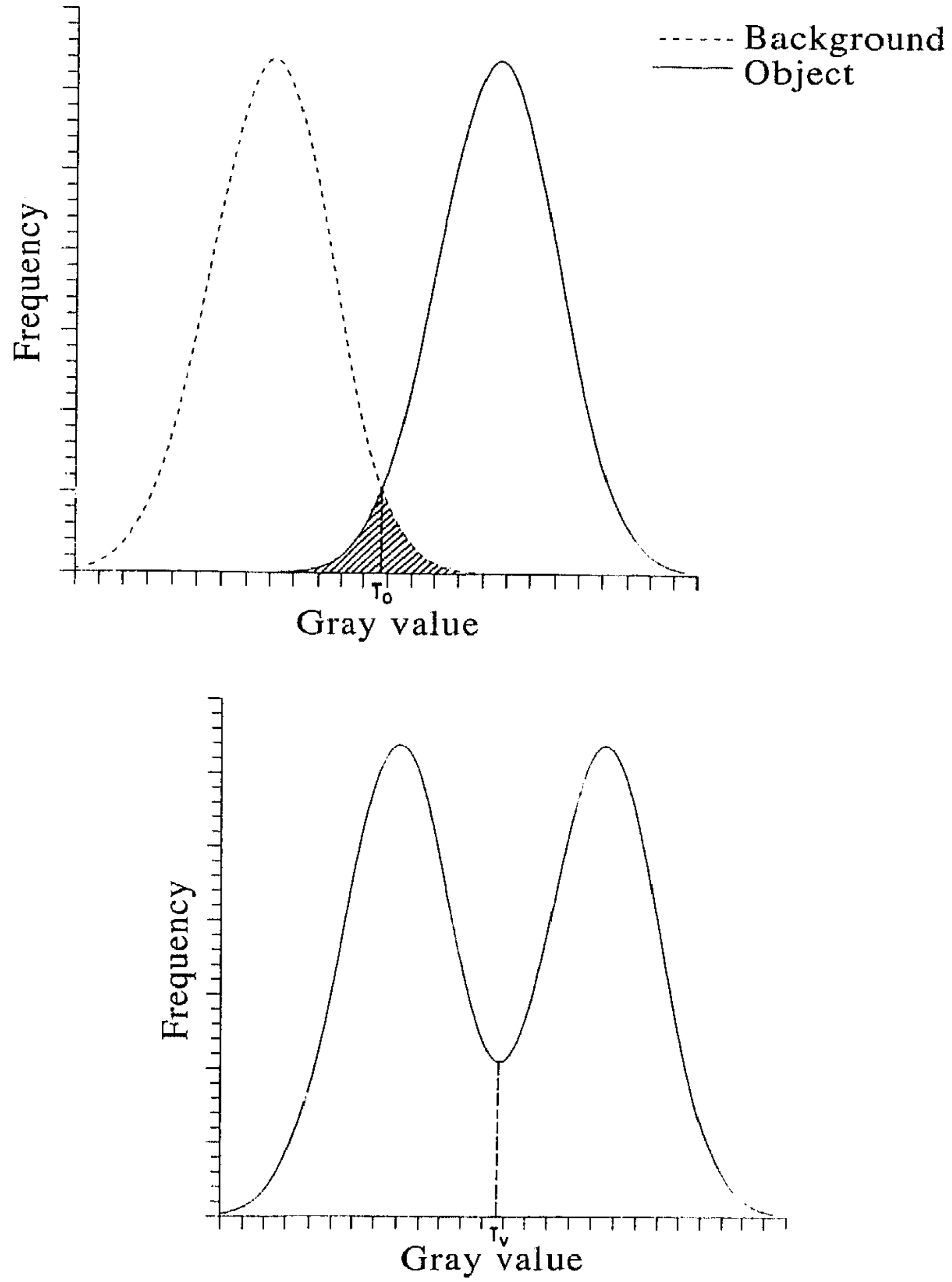

Fig. 1. (a) Object and background distributions with Baye's error shaded. (b) Resulting bimodal histogram.

histograms. This does not imply either that the histogram valleys always correspond to optimal threshold T_o or that the histograms generated by the model described in equation (3) are always bimodal. Glasbey[5] has observed a number of unimodal histograms in his experimental study, which also exist frequently in real life situations. We provide a possible reason behind this. In view of the above discussions, we now present the following lemma:

Lemma. Object and background distributions are guaranteed to generate a valley at T_v if

$$\left.\frac{\partial f}{\partial d}\right|_{d=d(v_1,T_v)} = \left.\frac{\partial f}{\partial d}\right|_{d=d(v_2,T_v)}, \quad \left.\frac{\partial^2 f}{\partial d^2}\right|_{d=d(T_v,v_i)} > 0, \quad i = 1, 2. \tag{5}$$

It is interesting to note here that the valley at T_v effectively coincides with the minimum error threshold at T_o, when the object and background regions have identical variances and sizes. Also, when f is a Gaussian function, it may be observed that a valley is guaranteed to be generated if $|v_i - T_v| > \sigma_i$, i.e. the Euclidian distance between the mean and the valley is more than the standard deviation.

The point of intersection of gray distribution $T_v \in \mathbf{L}$ is a valley if $h_{T_v^+} > h_{T_v}$ and $h_{T_v^-} > h_{T_v}$. This is possible if the rate at which background density falls at T_v^+ is less than the rate at which the object density increases and the rate at which the object density increases at T_v^- is less than the rate at which the background density falls. Each of these conditions, along with the continuity of partial derivatives, proves the lemma.

We also make the assumption that the gray density at the maximum and minimum gray values are negligibly small, i.e. $f(d(1, v_i), \sigma_i) < \varepsilon$ and $f(d(L, v_i), \sigma_i) < \varepsilon$, where ε is a small positive real number. This assumption allows us to estimate the parameters of the distribution more or less accurately.

2.2. *Thresholding based on k means algorithm*

The k means algorithm is one of the most popular unsupervised pattern classification schemes and is extensively applied in image analysis. It partitions the given set of samples into k classes by iteratively recomputing the partition. In each iteration, the means of all the classes are computed and a sample is assigned to a class corresponding to the minimum of the distances to various class means. Mathematically, the k means algorithm optimizes the sum of within cluster scatters and is guaranteed to provide accurate results when classes are compact and well separated.

In the case of thresholding, pixel classification can be done with the help of the k means algorithm. Otsu[6] minimized the within cluster scatter by varying the threshold T and identified the optimal threshold as the one which minimizes the scatter. Ridler[18] estimated an equivalent threshold with the help of an iterative k means algorithm, which assumes the following form:

Algorithm 1

1. Choose an initial threshold T_k.
2. Partition the image into two regions with the help of equation (1).
3. Compute the mean gray values, v_1 and v_2, of background and object regions, respectively.
4. Compute the new threshold $T_k = [(v_1 + v_2)/2]$.
5. Repeat steps 2–5 until there is no appreciable change for T_k.

It may be observed that when $\rho = \gamma = 1.0$, T_o coincided with T_k, which is equidistant from the means of the object and the background regions. When regions are not perfectly balanced, i.e. $\rho \neq 1$ or/and $\gamma \neq 1$, T_o is not guaranteed to be equal to T_k. It may be noted that T_k is equidistant from v_1 as well as v_2 and therefore Algorithm 1 is a favorable choice when regions are well balanced in size and scatter.

3. FUZZY THRESHOLDING USING FUZZY *c* MEANS

The above-described thresholding scheme segregates the object and background pixels in a deterministic fashion and the segmented description fails to reflect the structural details originally embedded in the gray distribution of the image. The fuzzy clustering algorithm provides possible solutions to accommodate the structural details in segmented description, since it identifies each of the clusters as a fuzzy set characterized by a membership distribution. The problem of fuzzy clustering is that of partitioning the set of n sample points $\mathbf{X} = \{x_1, x_2, \ldots, x_n\}$ into c classes such that the membership distribution has the following properties:

$$\mu_i(x_j) \in [0, 1], \tag{6a}$$

$$0 < \sum_{j=1}^{n} \mu_i(x_j) < n, \tag{6b}$$

and

$$\sum_{i=1}^{c} \mu_i(x_j) = 1.0. \tag{6c}$$

Objective function-based algorithms have been largely used for the identification of fuzzy partitions.[13] The fuzzy c means algorithm, a fuzzy extension of the k means algorithm, minimizes a least square objective function

$$J = \sum_{i=1}^{c} \sum_{j=1}^{n} \mu_i(x_j)^{\tau} d(x_j, v_i)^2 \tag{7}$$

to provide c hyper-spherical clusters. The parameter $\tau \geq 1$ controls the amount of fuzziness in the partition and when $\tau = 1$, the minima of equation (7) provides an equivalent classification as a k means algorithm.

Let $\{h_j\}$ be the histogram and $\{p_j\}$ be the probability distribution associated with the gray values of the image to be thresholded and $j \in \{0, 1, \ldots, L-1\}$. Thresholding may be formulated by modifying equation (7) with $c = 2$ as

$$J = \sum_{i=1}^{2} \sum_{j=0}^{L-1} h_j \mu_i(j)^{\tau} d(j, v_i)^2. \tag{8}$$

The cluster means for each class

$$v_i = \frac{\sum_{j=0}^{L-1} h_j j \mu_i(j)^{\tau}}{\sum_{j=0}^{L-1} h_j \mu_i(j)^{\tau}}, \quad i = 1, 2 \tag{9}$$

characterize the background and object regions. The objective function equation (8) can be iteratively minimized by computing the means with equation (9) and updating the memberships as

$$\mu_{\tilde{O}}(j) = \frac{1}{1 + \left[d(j, v_{\tilde{O}})/d(j, v_{\tilde{B}})\right]^{2/(\tau-1)}} \tag{10a}$$

and

$$\mu_{\tilde{B}}(j) = 1 - \mu_{\tilde{O}}(j). \tag{10b}$$

The fuzzy thresholding algorithm can be formally stated as follows:

Algorithm 2

1. Initialize the thresholded description $\mu_{\tilde{O}}$ and $\mu_{\tilde{B}}$ satisfying equation (6).
2. Compute the mean gray values of both the regions using equation (9).
3. Assign the membership values with equation (10).
4. Repeat steps 2–4 until there is no appreciable change for $\mu_{\tilde{O}}$ and $\mu_{\tilde{B}}$.

Even though the fuzzy c means algorithm is a generalization of the hard k means algorithm, the properties associated with the thresholded description-based on Algorithm 2 are quite different from that of Algorithm 1. An L gray value image can be represented with a set of L real values in thresholded description and reflect the geometry associated with the gray distribution. At the same time, the fuzzy thresholded description is more general, since a hard threshold, equivalent to the one obtained from k means, can be obtained from fuzzy thresholded description, as can be seen in the following theorem:

Theorem. (a) There exists one and only one gray value $T_f \in \mathbf{L}$ between v_1 and v_2 such that $\mu_{\tilde{O}}(T_f) = \mu_{\tilde{B}}(T_f)$.
(b) T_f is equidistant from v_1 and v_2.

From equation (10a), it may be seen that $\mu_{\tilde{O}}(v_{\tilde{B}}) = 0.0$ and $\mu_{\tilde{O}}(v_{\tilde{O}}) = 1.0$ and it is monotonically increasing in the range $[v_{\tilde{B}}, v_{\tilde{O}}]$. Similarly, $\mu_{\tilde{B}}$ is monotonically decreasing in the same range. This implies that there exist one and only one intersection of both the membership distributions, otherwise T_f is unique. From equation (10b), $\mu_{\tilde{B}}(T_f) = 0.5 = \mu_{\tilde{O}}(T_f)$, which also implies that $d(T_f, v_{\tilde{O}}) = d(T_f, v_{\tilde{B}})$, i.e. T_f is equidistant from both v_1 and v_2.

3.1. *On the hardening of fuzzy thresholded description*

Often our interest is restricted to extraction of the object from the scene so as to characterize the object with a set of features. Conventional object recognition methods may not be applicable as such with fuzzy thresholded description. Even though elegant image analysis techniques can be developed with fuzzy thresholded description, hardening schemes are required to make the description useful for classical object recognition schemes.

A simple hardening method may be with T_f, computed as described above and hardening with equation (1) to provide mutually exclusive and exhaustive object and background regions. Another possible hardening scheme may be based on α cuts of fuzzy sets as

$$O = \tilde{O}_\alpha = \{I_{mn} \mid \mu_{\tilde{O}}(I_{mn}) \geq \alpha\} \quad \text{and} \quad B = \mathscr{I} - O. \tag{11}$$

The parameter $\alpha \in [0, 1]$ controls the size of the object region. As α increases, O approaches the core/skeleton of the object region.

Proposition. There exists a corresponding hard threshold T only for $\alpha \in [\alpha_1, \alpha_2]$, so that the hardened description with equation (1) matches with equation (11), where

$$\alpha_1 = \frac{1}{1 + [d(0, v_2)/d(0, v_1)]^{2/(\tau-1)}} \quad \text{and}$$

$$\alpha_2 = \frac{1}{1 + [d(L-1, v_2)/d(L-1, v_1)]^{2/(\tau-1)}}.$$

Figure 2 depicts a typical fuzzy thresholded description for $\tau = 2.0$. Here $\alpha_1 = \mu_{\tilde{O}}(0)$ and $\alpha_2 = \mu_{\tilde{O}}(L-1)$. It may be seen that if $\alpha > \alpha_2$, pixels with gray values close to $L-1$ may be classified as background and for $\alpha < \alpha_1$, a set of pixels with gray values close to zero may be classified as object. In both cases the monotonicity is not preserved and the hardening process cannot be expressed with equation (1).

4. THRESHOLDING AS ESTIMATION OF NORMAL DENSITIES

The analytical procedure for selection of the optimal threshold can be simplified by assuming the object and background gray distributions to be normal. Kittler and

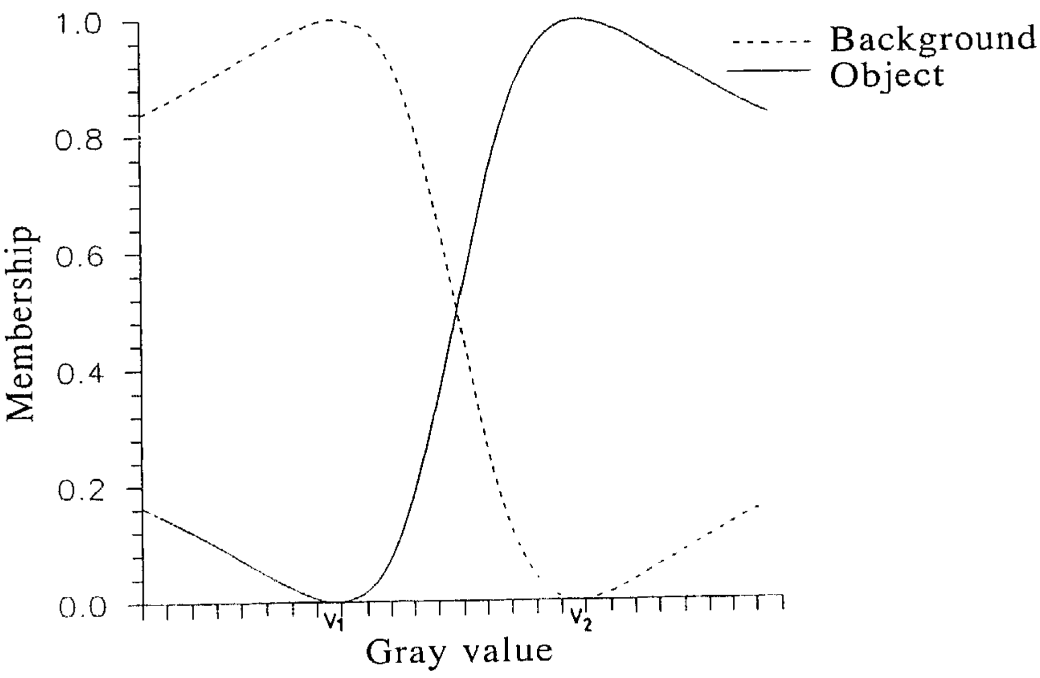

Fig. 2. A sample membership distribution with τ=2.0.

Illingworth[7] provide an elegant strategy for thresholding based on such an assumption. They have considered an objective function

$$J(T) = \sum_{j=0}^{T-1} p_j \left\{ \left(\frac{j - \nu_1}{\sigma_1}\right)^2 + 2\log\sigma_1 - 2\log P_1 \right\} + \sum_{j=T}^{L-1} p_j \left\{ \left(\frac{j - \nu_2}{\sigma_2}\right)^2 + 2\log\sigma_2 - 2\log P_2 \right\}, \quad (12)$$

where $P_1 = \sum_{j=0}^{T-1} p_j$ and $P_2 = \sum_{j=T}^{L-1} p_j$.

They argued that the global minima of J, i.e. $\min_{t\in \mathbf{L}} J(t)$, minimize the classification error and the corresponding threshold as the optimum. In various experimentations, with the help of normal assumptions, the algorithm is found to yield excellent results even though there are chances of identification of false thresholds on the extreme gray values in the presence of narrow vallies.

Our proposed thresholding scheme described in the previous section, however, does not assume the gray distribution to be normal. It provides the best results when regions are perfectly balanced in size as well as scatter, and generalizes Otsu's method since, T_f, which is always equidistant from the mean of object and background regions, corresponds to T_k. The problem of threshold selection in presence of region imbalances can be simplified to a great extent if we can transform the geometry of the gray values so that the optimum threshold is equidistant from the means of both the regions. This can be done by defining a suitable distance measure associated with each of the classes.

We define a pseudo-distance function

$$d(j, \nu_i) = \frac{1}{2}\left(\frac{j - \nu_i}{\sigma_i}\right)^2 + \log\sigma_i - \log\beta_i, \quad (13)$$

where β_i and σ_i are given by

$$\beta_i = \frac{\sum_{i=0}^{L-1} \mu_i^T(j) h_j}{\sum_{i=0}^{L-1} \mu_{\bar{O}}^T(j) h_j + \sum_{i=0}^{L-1} \mu_{\bar{B}}^T(j) h_j} \quad (14)$$

and

$$\sigma_i^2 = \frac{\sum_{j=0}^{L-1} \mu_i^T(j) h_j (j - \nu_i)^2}{\sum_{j=0}^{L-1} \mu_i^T(j) h_j}. \quad (15)$$

It may be noted that β_i is always less than or equal to 1 and β_i relates to ρ in the case of a hard partition as $\beta_1 = 1/(\rho + 1)$, $\beta_2 = \rho/(\rho + 1)$. With this distance measure, a gray value with equal density in object and background regions will be mapped as equidistant from both the region means. We thus present a modified fuzzy thresholding scheme below.

Algorithm 3

1. Initialize the thresholded description $\mu_{\bar{O}}$ and $\mu_{\bar{B}}$ satisfying equation (6).
2. Compute the mean gray values of both the regions using equation (9).
3. Compute β_i and σ_i, $i = 1, 2$ using equations (14) and (15).
4. Update the memberships using equations (10) and (13).
5. Repeat steps 2–5 until there is no appreciable change for $\mu_{\bar{O}}$ and $\mu_{\bar{B}}$.

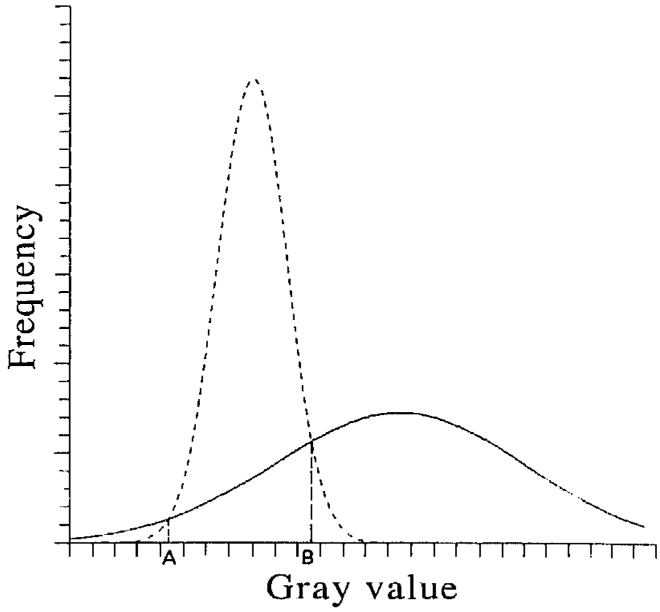

Fig. 3. Object and backgrund distributions, with the two gray values showing equal gray density.

The difference between the above algorithms and the iterative algorithm proposed by Kittler and Illingworth[7] may be brought out with the help of Fig. 3, where object and background gray densities are shown. The point of intersection of both the gray distributions may be unique or even multiple in **L**. The solution of a quadratic [reference (7), equation (21)] provides the two gray values having equal gray density, i.e. A and B in Fig. 3 and considers one of them as the new threshold. It may be seen that a pixel with gray value A or B in Fig. 3 may be an object or background pixel with equal possibility. That is, the membership of gray value A and B should be equal in object and background region. Our membership assignment scheme can provide such memberships as can be observed in equation (10). At the same time, iterative implementation of a minimum error threshold[7] considers one of the A or B as a new threshold and partitions in a hard fashion. The other pitfall of the algorithm, i.e. the problem of getting trapped in local minima, is also well handled in fuzzy algorithm, since fuzzy clustering algorithms are found to converge better due to the continuity of the partition spaces.[13] The fuzzy thresholded description available from Algorithm 3 may be hardened by any method described in the previous section. Here also it is guaranteed that there exist a hard gray value, T_f, in between ν_1 and ν_2 such that $\mu_{\bar{O}}(T_f) = \mu_{\bar{B}}(T_f)$.

5. RELAXING THE ASSUMPTIONS

Even though the assumption of a Gaussian distribution function is quite valid for many real life situations, the major reason behind the wide popularity of this assumption is its analytic tractability and the ease of parametrizing the distribution as well as its various higher order moments using the mean and standard deviation. The thresholding scheme described in the previous section depends extensively on this assumption and needs modification to handle a number of symmetrically decaying distributions occurring in real life situations.

As far as the threshold selection issues are concerned, Algorithm 2 is optimal if $\rho = \gamma = 1$. In general, when $\rho \neq 1$ and $\gamma \neq 10$ this algorithm may not yield proper results. Here we try to modify the histogram to incorporate the imbalance in size and the distance measure to accommodate the difference in standard deviations. Uncertainties due to the imprecision of gray values are incorporated in histograms[10] in fuzzy setting. A histogram incorporating the imbalance in the region sizes in a fuzzy environment may be defined as below.

Definition. A region normalized histogram $\{f_j\}$ is a set of real numbers computed with the help of the fuzzy membership distribution of the object and background regions as well as the histogram of the images. Specifically,

$$f_j = \frac{h_j \mu_{\tilde{B}}(j)}{\sum_{p=0}^{L-1} \mu_{\tilde{B}}(p)} + \frac{h_j \mu_{\tilde{O}}(j)}{\sum_{p=0}^{L-1} \mu_{\tilde{O}}(p)}. \tag{16}$$

The region normalized histogram is nothing but a normalized version of the original histogram with object and background regions showing approximately equal size.

Frequently in real life situations, γ is observed to be close to unity. In general, the assumption of $\gamma = 1$ may not be a valid one. The classical Mahalonobis distance in one dimension,

$$d(j, v_i) = \frac{|j - v_i|}{\sigma_i}, \tag{17}$$

can accommodate the variation in the dispersion of object and background regions, since it takes into account the class variances. In light of the above discussions, we present the fuzzy thresholding algorithm as follows:

Algorithm 4

1. Initialize the thresholded description $\mu_{\tilde{O}}$ and $\mu_{\tilde{B}}$ satisfying equation (6).
2. Compute the region normalized histogram with equation (16).
3. Compute the mean and standard deviation of gray values of both the regions with equations (9) and (15).
4. Update the memberships with equation (10) and the Mahalonobis distance function in one dimension with equation (17).
5. Repeat steps 2–5 until there is no appreciable change for $\mu_{\tilde{O}}$ and $\mu_{\tilde{B}}$.

The basic fuzzy c means algorithm is found to converge fast and is guaranteed to provide an optimal fuzzy partition.[13] The above described algorithm, resembling the FCM algorithm, also converges fast. In each of the iterations, however, the histogram is modified to provide a pseudo-gray distribution such that object and backgrounds have equal sizes. When the geometry extracted becomes complicated, convergence is accelerated with good initialization. A few iterations of Algorithm 2 may be a good initialization for Algorithm 4.

6. DISCUSSIONS

The classical thresholding schemes assign the pixel unequivocally and do not distinguish among pixels in a region even if their gray values are different in the original image. Consequently, the hard threshold selection schemes are associated with loss of structural details on thresholding. The identities of pixels are preserved in fuzzy partition space, since the memberships assigned to the pixels depend on the difference between its gray value and the mean gray value of the region to which it belongs. The other major difference between the proposed algorithm and the popular ones is in its implementation with iterative formulations. Many algorithms, instead of optimizing iteratively, go on an extensive search to identify the global maxima or minima associated with the objective function. In general, the algorithms discussed in this paper are efficient and are superior to the hard algorithms reported in literature.

The error of classification in the thresholding process is defined as the number of pixels misclassified during thresholding, i.e. the sum of the number of object pixels with a gray value less than T and the number of background pixels with a gray value greater than or equal to T. Comparison of fuzzy and hard thresholding schemes is quite difficult since the concept of deterministic classification and therefore misclassification are associated only with hard thresholds. Even then a preliminary comparison may be carried out with the help of the hard threshold(T_f), obtained from a fuzzy partition.

To validate the applicability of the proposed fuzzy thresholding algorithms, we provide experimental results on synthetic images and compare the performance based on classification error. Synthetic images of size 256×256 pixels are generated for various values of ρ and γ. Object and background regions are assumed to be normal with means of 80 and 175, respectively. Bayes's classification, the optimal minimum error classification, is compared with various hard and fuzzy threshold selection schemes. Other hard algorithms considered are the minimum error threshold,[7] Otsu,[6] moments,[9] and Kapur.[8] Fuzzy thresholding algorithms based on fuzzy clustering are decomposed into hard by identifying a hard gray value between the mean gray value of object and background regions with equal membership in both the regions. All the thresholds are approximated to the nearest integer and along with the corresponding classification errors are provided in Tables 1 and 2. It may be seen that the fuzzy thresholding schemes based on fuzzy clustering are performing well for various reasonable object and background imbalances. Even after decomposing into hard descriptions they are found to perform well and thus validate the generality and superiority of the fuzzy algorithms for thresholding.

Table 1. Comparative performance of various hard thresholding schemes in the presence of mixed normal densities

Parameters			Threshold/error				
σ_1	σ_2	ρ	Bayes	Kittler	Otsu	Moment	Kapur
15	15	1.00	128/50.425	128/50.425	127/51.649	127/51.649	129/51.649
15	15	0.50	130/47.440	130/47.440	127/55.529	107/1687.2	106/1945.8
15	15	0.33	131/43.273	131/43.273	127/57.469	105/2515.0	106/2189.1
15	5	1.00	151/0.0575	151/0.0575	127/31.645	103/2187.9	89/9353.12
15	10	1.00	137/4.6450	137/4.6450	127/31.665	107/1265.3	105/1676.7
15	10	0.50	138/4.6716	138/4.6716	127/42.206	104/2558.6	101/3749.8
15	10	0.33	139/4.4920	139/4.4920	127/47.477	103/3281.9	102/3727.9
15	5	0.33	152/0.0663	151/0.0711	127/47.468	102/3727.9	94/9044.74
15	5	2.00	150/0.0462	150/0.0462	127/21.096	104/1279.3	163/268.15
15	5	3.00	150/0.0371	149/0.0428	127/15.703	105/832.03	163/302.40
15	10	3.00	135/3.5200	135/3.5200	127/15.730	151/350.98	156/1258.5
15	10	2.00	136/4.0440	136/4.0440	127/21.123	117/163.21	155/879.96

Table 2. Comparative performance of fuzzy thresholding schemes in the presence of mixed normal densities

Parameters			Threshold/error				
σ_1	σ_2	ρ	Pal	Huang	Alg.2	Alg. 3	Alg. 4
15	15	1.00	127/51.649	128/50.425	128/50.425	127/51.649	128/50.425
15	15	0.50	132/53.680	128/50.425	127/55.529	129/47.760	127/55.520
15	15	0.33	135/63.570	128/50.425	127/57.469	129/45.829	126/67.230
15	5	1.00	130/15.806	128/25.212	127/31.645	151/0.0575	153/0.1293
15	10	1.00	129/20.058	128/25.246	127/31.665	137/4.6250	138/4.9445
15	10	0.50	133/10.370	128/33.638	127/42.206	138/4.6716	138/4.6716
15	10	0.33	135/7.2740	128/37.830	127/47.477	139/4.4920	138/4.5370
15	5	0.33	136/5.2840	128/37.819	127/47.468	152/0.0663	153/0.0866
15	5	2.00	128/16.808	128/16.808	127/21.096	150/0.0462	153/0.1579
15	5	3.00	126/19.629	128/12.511	127/15.703	150/0.0370	153/0.1724
15	10	3.00	123/37.390	128/12.560	127/15.730	136/3.6600	139/7.2020
15	10	2.00	125/32.830	128/16.850	127/21.123	136/4.0400	139/6.7420

7. CONCLUSION

The problem of pixel classification is well suited to be formulated as a clustering problem. The conventional valley searching threshold selection schemes need not provide the optimal threshold in a pattern recognition sense. The classical deterministic partitions do not reflect the geometry of the optimal gray distribution, but this limitation is taken care of in a fuzzy environment. The problem of thresholding in the presence of region imbalances is analyzed by transforming the geometrical structure of gray distribution by modeling the distance and density function. Analytical discussions and experimental details validate the importance of fuzzy thresholding schemes based on fuzzy clustering.

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